2013-14 CC MathCourse1 BK (B195911)—Blueprint Summary

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				2013-2014 CC		2013-2014 CC	
			Status	Draft		Draft	
			# Standards Assessed	11		12	
			Number of Items per Standard (max)	2		2	
			Number of Items per Standard (min)	1		1	
			Number of Items per Standard (avg)	1.5		1.3	
Standard	**Description**	**Yr #**	**Yr %**	**#**	**%**	**#**	**%**
Total		**32**	**100%**	**16**	**100%**	**16**	**100%**
Common Core Item Bank							
Introduction	Introduction						
During the years fr	During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, “number” means “counting number”: 1, 2, 3... Soon after that, 0 is used to represent “none” and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.						
With each extensic	With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.						
Extending the prop	Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that (5 to the 1/3 power)^3 should be 5 to the $(1/3)^3$ power $= 5^1 = 5$ and that 5 to the 1/3 power should be the cube root of 5.						

Calculators, spread	Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.						
In real world proble	In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.						
MA.9-12.N-RN	The Real Number System						
MA.9-12.N-RN.A	Extend the properties of exponents to rational exponents.						
MA.9-12.N-RN.A.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.						
MA.9-12.N-RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.						
MA.9-12.N-RN.B	Use properties of rational and irrational numbers.						
MA.9-12.N-RN.B.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.						
MA.9-12.N-Q	Quantities						
MA.9-12.N-Q.A	Reason quantitatively and use units to solve problems.						

MA.9-12.N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.						
MA.9-12.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.						
MA.9-12.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.						
MA.9-12.N-CN	The Complex Number System						
MA.9-12.N-CN.A	Perform arithmetic operations with complex numbers.						
MA.9-12.N-CN.A.1	Know there is a complex number i such that $i^2 = –1$, and every complex number has the form a + bi with a and b real.						
MA.9-12.N-CN.A.2	Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.						
MA.9-12.N-CN.A.3	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.						
MA.9-12.N-CN.B	Represent complex numbers and their operations on the complex plane.						
MA.9-12.N-CN.B.4	Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.						
MA.9-12.N-CN.B.5	Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.						
MA.9-12.N-CN.B.6	Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.						
MA.9-12.N-CN.C	Use complex numbers in polynomial identities and equations.						
MA.9-12.N-CN.C.7	Solve quadratic equations with real coefficients that have complex solutions.						
MA.9-12.N-CN.C.8	Extend polynomial identities to the complex numbers.						
MA.9-12.N-CN.C.9	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.						
MA.9-12.N-VM	Vector and Matrix Quantities						
MA.9-12.N-VM.A	Represent and model with vector quantities.						
MA.9-12.N-VM.A.1	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v).						
MA.9-12.N-VM.A.2	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.						
MA.9-12.N-VM.A.3	Solve problems involving velocity and other quantities that can be represented by vectors.						

MA.9-12.N-VM.B	Perform operations on vectors.						
MA.9-12.N-VM.B.4	Add and subtract vectors.						
MA.9-12.N-VM.B.4	Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.						
MA.9-12.N-VM.B.4	Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.						
MA.9-12.N-VM.B.4	Understand vector subtraction v – w as v + (–w), where –w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.						
MA.9-12.N-VM.B.5	Multiply a vector by a scalar.						
MA.9-12.N-VM.B.5	Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(v subscript x, v subscript y) = (cv subscript x, cv subscript y).						
MA.9-12.N-VM.B.5	Compute the magnitude of a scalar multiple cv using ||cv|| = |c|v. Compute the direction of cv knowing that when |c|v is not equal to 0, the direction of cv is either along v (for c > 0) or against v (for c < 0).						
MA.9-12.N-VM.C	Perform operations on matrices and use matrices in applications.						
MA.9-12.N-VM.C.6	Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.						
MA.9-12.N-VM.C.7	Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.						
MA.9-12.N-VM.C.8	Add, subtract, and multiply matrices of appropriate dimensions.						
MA.9-12.N-VM.C.9	Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.						
MA.9-12.N-VM.C.1	Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.						
MA.9-12.N-VM.C.1	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.						
MA.9-12.N-VM.C.1	Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.						
Introduction	Introduction						

An expression is a	An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.						
Reading an expres	Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, p + 0.05p can be interpreted as the addition of a 5% tax to a price p. Rewriting p + 0.05p as 1.05p shows that adding a tax is the same as multiplying the price by a constant factor.						
Algebraic manipula	Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, p + 0.05p is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.						
A spreadsheet or a	A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.						
An equation is a sta	An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.						
The solutions of an	The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.						

An equation can of	An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.						
Some equations ha	Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of x + 1 = 0 is an integer, not a whole number; the solution of 2x + 1 = 0 is a rational number, not an integer; the solutions of $x^2 – 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.						
The same solution	The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, A = ((b1+b2)/2)h, can be solved for h using the same deductive process.						
Inequalities can be	Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.						
Expressions can de	Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.						
MA.9-12.A-SSE	Seeing Structure in Expressions						
MA.9-12.A-SSE.A	Interpret the structure of expressions						
MA.9-12.A-SSE.A.	Interpret expressions that represent a quantity in terms of its context.						
MA.9-12.A-SSE.A.	Interpret parts of an expression, such as terms, factors, and coefficients.						
MA.9-12.A-SSE.A.	Interpret complicated expressions by viewing one or more of their parts as a single entity.						
MA.9-12.A-SSE.A.	Use the structure of an expression to identify ways to rewrite it.						
MA.9-12.A-SSE.B	Write expressions in equivalent forms to solve problems						
MA.9-12.A-SSE.B.	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.						
MA.9-12.A-SSE.B.	Factor a quadratic expression to reveal the zeros of the function it defines.						
MA.9-12.A-SSE.B.	Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.						
MA.9-12.A-SSE.B.	Use the properties of exponents to transform expressions for exponential functions.						

MA.9-12.A-SSE.B.4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.						
MA.9-12.A-APR	Arithmetic with Polynomials and Rational Expressions						
MA.9-12.A-APR.A	Perform arithmetic operations on polynomials						
MA.9-12.A-APR.A.	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.						
MA.9-12.A-APR.B	Understand the relationship between zeros and factors of polynomials						
MA.9-12.A-APR.B.	Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).						
MA.9-12.A-APR.B.	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.						
MA.9-12.A-APR.C	Use polynomial identities to solve problems						
MA.9-12.A-APR.C.	Prove polynomial identities and use them to describe numerical relationships.						
MA.9-12.A-APR.C.	Know and apply the Binomial Theorem for the expansion of (x + y) to the n power in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal’s Triangle.						
MA.9-12.A-APR.D	Rewrite rational expressions						
MA.9-12.A-APR.D.	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.						
MA.9-12.A-APR.D.	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.						
MA.9-12.A-CED	Creating Equations						
MA.9-12.A-CED.A	Create equations that describe numbers or relationships						
MA.9-12.A-CED.A.	Create equations and inequalities in one variable and use them to solve problems.	1	3.1%			1	6.3%
MA.9-12.A-CED.A.	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	3	9.4%	2	12.5%	1	6.3%
MA.9-12.A-CED.A.	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.						

MA.9-12.A-CED.A.	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.						
MA.9-12.A-REI	Reasoning with Equations and Inequalities						
MA.9-12.A-REI.A	Understand solving equations as a process of reasoning and explain the reasoning	1	3.1%			1	6.3%
MA.9-12.A-REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	2	6.3%			2	12.5%
MA.9-12.A-REI.A.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.						
MA.9-12.A-REI.B	Solve equations and inequalities in one variable						
MA.9-12.A-REI.B.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	1	3.1%			1	6.3%
MA.9-12.A-REI.B.4	Solve quadratic equations in one variable.						
MA.9-12.A-REI.B.4	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.						
MA.9-12.A-REI.B.4	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.						
MA.9-12.A-REI.C	Solve systems of equations						
MA.9-12.A-REI.C.5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.						
MA.9-12.A-REI.C.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	2	6.3%			2	12.5%
MA.9-12.A-REI.C.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.						
MA.9-12.A-REI.C.8	Represent a system of linear equations as a single matrix equation in a vector variable.						
MA.9-12.A-REI.C.9	Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).						
MA.9-12.A-REI.D	Represent and solve equations and inequalities graphically						
MA.9-12.A-REI.D.1	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	2	6.3%	2	12.5%		

MA.9-12.A-REI.D.1	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.						
MA.9-12.A-REI.D.1	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	1	3.1%			1	6.3%
Introduction	Introduction						
Functions describe	Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.						
In school mathema	In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car’s speed in miles per hour, v; the rule T(v) = 100/v expresses this relationship algebraically and defines a function whose name is T.						
The set of inputs to	The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.						
A function can be d	A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, “I’ll give you a state, you give me the capital city;” by an algebraic expression like f(x) = a + bx; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function’s properties.						
Functions presente	Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.						

A graphing utility or	A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.						
Determining an out	Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.						
MA.9-12.F-IF	Interpreting Functions						
MA.9-12.F-IF.A	Understand the concept of a function and use function notation						
MA.9-12.F-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).	1	3.1%	1	6.3%		
MA.9-12.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	2	6.3%	2	12.5%		
MA.9-12.F-IF.A.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.						
MA.9-12.F-IF.B	Interpret functions that arise in applications in terms of the context	1	3.1%	1	6.3%		
MA.9-12.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.	2	6.3%	2	12.5%		
MA.9-12.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.	1	3.1%	1	6.3%		
MA.9-12.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	1	3.1%	1	6.3%		
MA.9-12.F-IF.C	Analyze functions using different representations						
MA.9-12.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.						
MA.9-12.F-IF.C.7.a	Graph linear and quadratic functions and show intercepts, maxima, and minima.						
MA.9-12.F-IF.C.7.b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.						

MA.9-12.F-IF.C.7.c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.						
MA.9-12.F-IF.C.7.d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.						
MA.9-12.F-IF.C.7.e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.						
MA.9-12.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.						
MA.9-12.F-IF.C.8.a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.						
MA.9-12.F-IF.C.8.b	Use the properties of exponents to interpret expressions for exponential functions.						
MA.9-12.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).						
MA.9-12.F-BF	Building Functions						
MA.9-12.F-BF.A	Build a function that models a relationship between two quantities						
MA.9-12.F-BF.A.1	Write a function that describes a relationship between two quantities.						
MA.9-12.F-BF.A.1.	Determine an explicit expression, a recursive process, or steps for calculation from a context.						
MA.9-12.F-BF.A.1.	Combine standard function types using arithmetic operations.						
MA.9-12.F-BF.A.1.	Compose functions.						
MA.9-12.F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.						
MA.9-12.F-BF.B	Build new functions from existing functions						
MA.9-12.F-BF.B.3	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.						
MA.9-12.F-BF.B.4	Find inverse functions.						
MA.9-12.F-BF.B.4.	Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.						
MA.9-12.F-BF.B.4.	Verify by composition that one function is the inverse of another.						
MA.9-12.F-BF.B.4.	Read values of an inverse function from a graph or a table, given that the function has an inverse.						
MA.9-12.F-BF.B.4.	Produce an invertible function from a non-invertible function by restricting the domain.						
MA.9-12.F-BF.B.5	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.						

MA.9-12.F-LE	Linear, Quadratic, and Exponential Models						
MA.9-12.F-LE.A	Construct and compare linear, quadratic, and exponential models and solve problems						
MA.9-12.F-LE.A.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.						
MA.9-12.F-LE.A.1.	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.						
MA.9-12.F-LE.A.1.	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.						
MA.9-12.F-LE.A.1.	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.						
MA.9-12.F-LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).						
MA.9-12.F-LE.A.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.						
MA.9-12.F-LE.A.4	For exponential models, express as a logarithm the solution to ab to the ct power = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.						
MA.9-12.F-LE.B	Interpret expressions for functions in terms of the situation they model						
MA.9-12.F-LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context.						
MA.9-12.F-TF	Trigonometric Functions						
MA.9-12.F-TF.A	Extend the domain of trigonometric functions using the unit circle						
MA.9-12.F-TF.A.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.						
MA.9-12.F-TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.						
MA.9-12.F-TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for pi/3, pi/4 and pi/6, and use the unit circle to express the values of sine, cosine, and tangent for pi–x, pi+x, and 2pi–x in terms of their values for x, where x is any real number.						
MA.9-12.F-TF.A.4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.						
MA.9-12.F-TF.B	Model periodic phenomena with trigonometric functions						
MA.9-12.F-TF.B.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.						

MA.9-12.F-TF.B.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.						
MA.9-12.F-TF.B.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.						
MA.9-12.F-TF.C	Prove and apply trigonometric identities						
MA.9-12.F-TF.C.8	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(theta), cos(theta), or tan(theta) given sin(theta), cos(theta), or tan(theta) and the quadrant of the angle.						
MA.9-12.F-TF.C.9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.						
Introduction	Introduction						
Modeling links clas	Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.						
A model can be ve	A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.						
Some examples of	Some examples of such situations might include:						
Estimating how mu	Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.						
Planning a table te	Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.						

Designing the layo	Designing the layout of the stalls in a school fair so as to raise as much money as possible.						
Analyzing stopping	Analyzing stopping distance for a car.						
Modeling savings a	Modeling savings account balance, bacterial colony growth, or investment growth.						
Engaging in critical	Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.						
Analyzing risk in si	Analyzing risk in situations such as extreme sports, pandemics, and terrorism.						
Relating populatior	Relating population statistics to individual predictions.						
In situations like th	In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.						
One of the insights	One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.						
The basic modelin	The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.						

In descriptive mode	In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model— for example, graphs of global temperature and atmospheric CO2 over time.						
Analytic modeling s	Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.						
Graphing utilities, s	Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.						
Modeling is best in	Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (?).						
MA.9-12.M	Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol.						
Introduction	Introduction						
An understanding o	An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.						
Although there are	Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)						
During high school	During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.						

The concepts of co	The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.							
In the approach tak	In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.							
Similarity transform	Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.							
The definitions of s	The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to nonright triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.							

Analytic geometry c	Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.						
Dynamic geometry	Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.						
The correspondenc	The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.						
MA.9-12.G-CO	Congruence						
MA.9-12.G-CO.A	Experiment with transformations in the plane						
MA.9-12.G-CO.A.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.						
MA.9-12.G-CO.A.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	1	3.1%	1	6.3%		
MA.9-12.G-CO.A.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	1	3.1%	1	6.3%		
MA.9-12.G-CO.A.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.						

MA.9-12.G-CO.A.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	2	6.3%	2	12.5%		
MA.9-12.G-CO.B	Understand congruence in terms of rigid motions						
MA.9-12.G-CO.B.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	2	6.3%			2	12.5%
MA.9-12.G-CO.B.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	1	3.1%			1	6.3%
MA.9-12.G-CO.B.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	2	6.3%			2	12.5%
MA.9-12.G-CO.C	Prove geometric theorems						
MA.9-12.G-CO.C.9	Prove theorems about lines and angles.						
MA.9-12.G-CO.C.1	Prove theorems about triangles.						
MA.9-12.G-CO.C.1	Prove theorems about parallelograms.						
MA.9-12.G-CO.D	Make geometric constructions						
MA.9-12.G-CO.D.1	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).						
MA.9-12.G-CO.D.1	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.						
MA.9-12.G-SRT	Similarity, Right Triangles, and Trigonometry						
MA.9-12.G-SRT.A	Understand similarity in terms of similarity transformations						
MA.9-12.G-SRT.A.	Verify experimentally the properties of dilations given by a center and a scale factor:						
MA.9-12.G-SRT.A.	A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.						
MA.9-12.G-SRT.A.	The dilation of a line segment is longer or shorter in the ratio given by the scale factor.						
MA.9-12.G-SRT.A.	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.						
MA.9-12.G-SRT.A.	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.						
MA.9-12.G-SRT.B	Prove theorems involving similarity						
MA.9-12.G-SRT.B.	Prove theorems about triangles.						

MA.9-12.G-SRT.B.	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.						
MA.9-12.G-SRT.C	Define trigonometric ratios and solve problems involving right triangles						
MA.9-12.G-SRT.C.	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.						
MA.9-12.G-SRT.C.	Explain and use the relationship between the sine and cosine of complementary angles.						
MA.9-12.G-SRT.C.	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.						
MA.9-12.G-SRT.D	Apply trigonometry to general triangles						
MA.9-12.G-SRT.D.	Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.						
MA.9-12.G-SRT.D.	Prove the Laws of Sines and Cosines and use them to solve problems.						
MA.9-12.G-SRT.D.	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).						
MA.9-12.G-C	Circles						
MA.9-12.G-C.A	Understand and apply theorems about circles						
MA.9-12.G-C.A.1	Prove that all circles are similar.						
MA.9-12.G-C.A.2	Identify and describe relationships among inscribed angles, radii, and chords.						
MA.9-12.G-C.A.3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.						
MA.9-12.G-C.A.4	Construct a tangent line from a point outside a given circle to the circle.						
MA.9-12.G-C.B	Find arc lengths and areas of sectors of circles						
MA.9-12.G-C.B.5	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.						
MA.9-12.G-GPE	Expressing Geometric Properties with Equations						
MA.9-12.G-GPE.A	Translate between the geometric description and the equation for a conic section						
MA.9-12.G-GPE.A.	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.						
MA.9-12.G-GPE.A.	Derive the equation of a parabola given a focus and directrix.						
MA.9-12.G-GPE.A.	Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.						
MA.9-12.G-GPE.B	Use coordinates to prove simple geometric theorems algebraically						
MA.9-12.G-GPE.B.	Use coordinates to prove simple geometric theorems algebraically.						

MA.9-12.G-GPE.B.	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	1	3.1%			1	6.3%
MA.9-12.G-GPE.B.	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.						
MA.9-12.G-GPE.B.	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	1	3.1%			1	6.3%
MA.9-12.G-GMD	Geometric Measurement and Dimension						
MA.9-12.G-GMD.A	Explain volume formulas and use them to solve problems						
MA.9-12.G-GMD.A	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.						
MA.9-12.G-GMD.A	Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.						
MA.9-12.G-GMD.A	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.						
MA.9-12.G-GMD.B	Visualize relationships between two-dimensional and three-dimensional objects						
MA.9-12.G-GMD.B	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.						
MA.9-12.G-MG	Modeling with Geometry						
MA.9-12.G-MG.A	Apply geometric concepts in modeling situations						
MA.9-12.G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).						
MA.9-12.G-MG.A.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).						
MA.9-12.G-MG.A.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).						

Key:
Orange: This standard has 6 or more items on the assessment.
Purple: This standard has 26 or more items on the blueprint.